EYE MOVEMENTS DURING GEOMETRY PROOF READING: TEXT CONTRASTING WITH FIGURE AND THE COLOR EFFECTS

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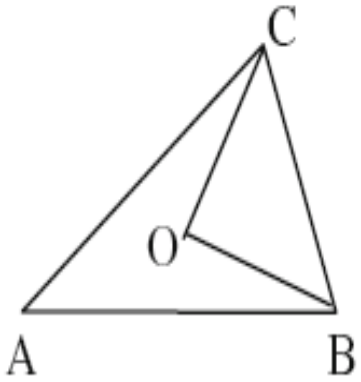
Introduction

- Proofs are viewed as a genre of mathematical text , but there are many special skills of reading comprehension in geometry proofs.
- Eye-tracking technique has been used in reading comprehension studies.
- Color figures improved reading comprehension in textbook.

Cognitive process of geometry study and reading theories of proof

- Researchers have long reported the complex cognitive process of geometry (e.g.,Barton, Heidema, & Jordan, 2002；Adams, 2003) and explored a reading comprehension of geometry proof model (RCGP) (Yang & Lin ,2008)
- Duval (1995, 1999) distinguished four apprehensions for a “geometrical figure”
- Gal & Linchevski（2010) used visual perception and perception-based knowledge representation (VPR) to explain difficulties encountered in figural processing.



O is the outer center* of triangle ABC.
Show that the angle BOC is twice the same as angle BAC.

*In Chinese, we call the center of circumcircle of a triangle as 'outer center'.

Fig. 3 Chang and Lin's (2005) assignment

Eye movement researches about text and diagram reading

- First-fixation duration (FD) or gaze duration (GD) represents early word recognition processes.

  The number of regressions was used as measures of reanalysis (Juhasz & Rayner,2003; Williams & Morris,2004)
- In scientific text, Hegarty (1992, 1995) ,Schmidt-Weigand，Kohnert & Glowalla（2010) , Hannus & Hyona（1999) : reading comprehension in multimedia is largely text directed .

  The percentages of fixation in diagrams of these researches are about 0.07~0.3

The main questions of this study

- What's the percentages of fixation in figures during geometry proofs reading?
- What are the color effects of figures on eye movements during geometry proofs reading?
- What are the color effects of figures on paper-pencil recall post-tests after geometry proofs reading?

Method-Participants

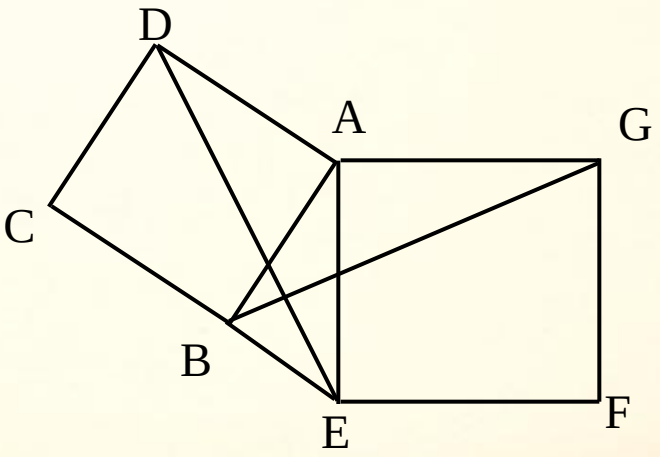
- 35 undergraduate students
- with normal or corrected-to-normal vision
- had learned geometry proofs but were not familiar with these proofs now.

Method-Materials and design

- Four junior high school level geometry proof problems
  - Proof with colored or uncolored figures ex.

已知：如右圖，ABCD、AEFG均為正方形。

求證：$\triangle ADE \cong \triangle ABG$



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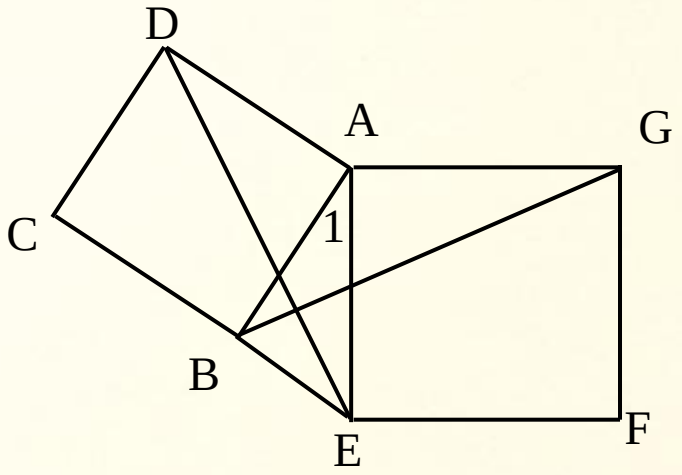

證明：

1. $\because$ ABCD、AEFG均為正方形

   $\therefore \quad \overline{AB}=\overline{AD}$ ，$\overline{AE}=\overline{AG}$

2. $\because \quad \angle DBA=90^0=\angle GAE$

   $\therefore \quad \angle EAD=\angle BAD+\angle 1=\angle GAE+\angle 1=\angle GAB$

   故 $\triangle ADE \cong \triangle ABG$（SAS全等）

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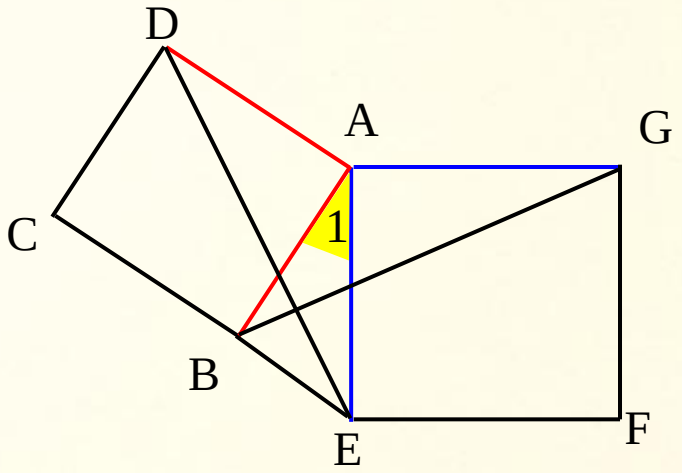

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Method-Materials and design

- Four junior high school level geometry proof problems
  - Proof with colored or uncolored figures ex.
- Paper-and-pencil recall post-test

Method-Apparatus

- Eyelink 1000 eyetracker
- sampling frequency:1000 Hz
- 19-inch monitor

  maximum resolution 1280 ×1024 pixels
- Thresholds :

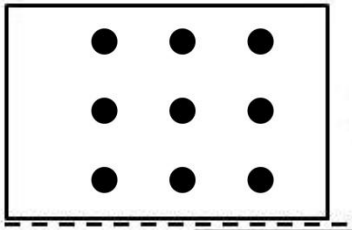
  Motion (0.2deg)

  velocity (30/s)

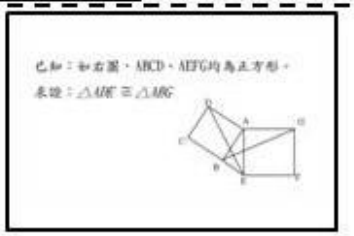
  Acceleration(8000deg/ s^2).

§ Procedures

Eye movement experiment

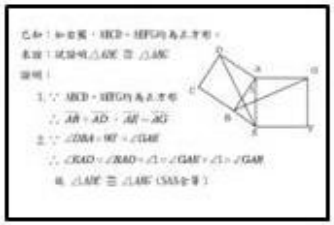


calibration

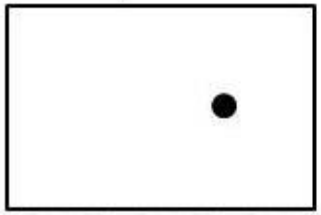


item(25 seconds)

four items



worked proof



calibration

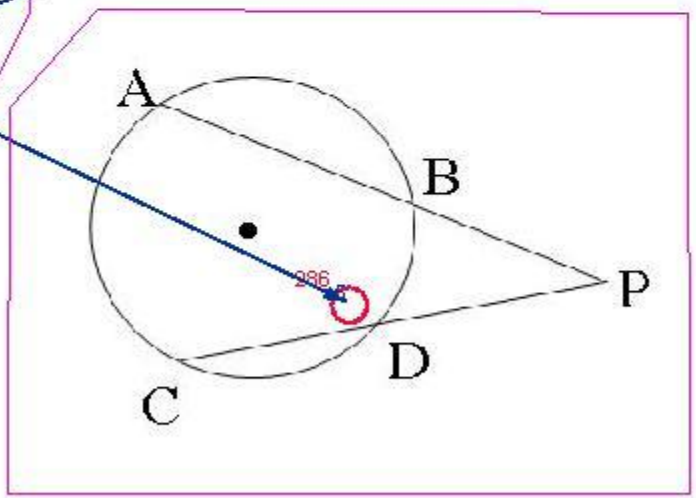
Paper-pencil recallpost test

Method-data analysis（1）

- Text contrasting with figure : 31 participants
- After exclusion the drifting data, color effect on critical propositions : 19~26
- Exclude nonsense fixations.
  - Fixation duration < 100 msec
  - Other nonsense fixations

已知：如右圖，兩弦 $\overline{AB}$ 與 $\overline{CD}$ 的延長線交於圓外一點P

求證：$\angle P = \frac{1}{2}\left(\overset{\frown}{AC} - \overset{\frown}{BD}\right)$



Method-data analysis （2）

- Two kinds of eye movement data:

  initial reading comprehension time: first gaze duration (GD)

  regression time : regressive gaze (RG)

Fixation numbers 1 to 7 indicate first gaze duration

已 BCD、AEFG均為正方形。

求 $\triangle ADE \cong \triangle ABG$

證明

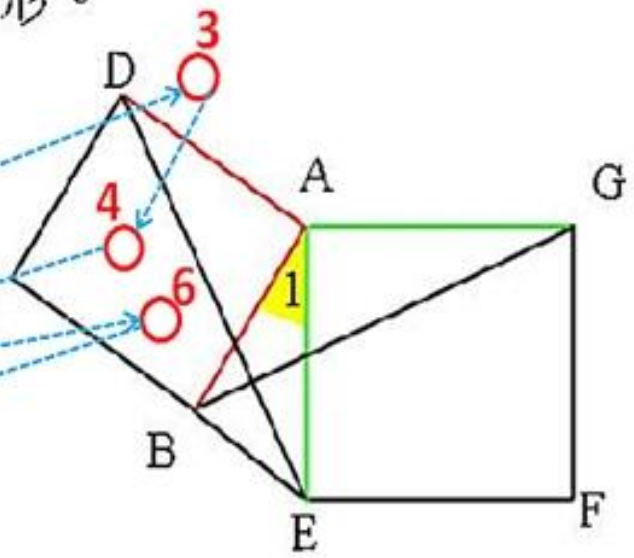
1. ∵ ABCD、AEFG均為正方形

∴ $\overline{AB} = \overline{AD}$, $\overline{AE} = \overline{AG}$

2. ∵ $\angle DAB = 90^0 - \angle G$

∴ $\angle EAD = \angle BAD$

故 $\triangle ADE \cong \triangle ABG$ (SAS全等)



If there were any fixation locates backwards in $\overline{AB} = \overline{AD}$ after 8, those will be regressive gaze

Figure 1 Example of the initial reading comprehension time

Method-data analysis （3）

- Paper-pencil recall post-test

  No.1~3: 5 grades

  No.4: 9 grades

Result (1)

Table1　Fixation on Figures and texts in four problems

	Item 1	Item 2	Item 3	Iterm 4
Problem				
Fixation duration-figure(sec)	12.40	8.18	10.54	9.06
Fixation duration-text(sec)	20.47	19.90	20.17	19.67
rate of fixation on figure(%)	61	41	52	46
Proof				
Fixation duration-figure(sec)	20.73	15.68	19.56	48.06
Fixation duration-text(sec)	20.53	25.95	24.37	63.32
Fixation duration-total	41.26	41.43	43.93	111.38
rate of fixation on figure(%)	48	38	44	43

percentages of fixation duration on figure are about 38%~61%

Result (2)

Table2　Color effect on fixations on proofs

color	figure	text	total	*F*
No.1				
color (*n*=16)	21.10	19.71	40.81	2
uncolor (*n*=15)	20.33	21.39	41	
No.2				
color (*n*=15)	14.91	23.82	38.	
uncolor (*n*=16)	16.40	27.94	44.3	
No.3				
color (*n*=16)	20.95	28.18	49.13	
uncolor (*n*=15)	18.08	20.30	38.	
No.4				
color (*n*=15)	38.27	56.91	95.18	4.45*
uncolor (*n*=16)	57.23	69.33	126.56	

Total fixation of color effect on critical theorems of No.4 was significant

Result (3)

Table3 Total fixation, initial reading comprehension times & regression time of color effect on critical propositions

color	Total fixation	Initial reading comprehension times	Regression time
No.1			
color (*n*=10)	21.12	**8.30**[a]	
uncolor (*n*=10)			
No.2			
color (*n*=11)			
uncolor (*n*=15)			
No.3			
color (*n*=11)			
uncolor (*n*=15)		3.16	16.66
No.4			
color (*n*=12)	**42.60**	**4.22**	38.38
uncolor (*n*=11)	**59.79**	**8.66**	51.12

Total fixation c color eff on criti propos of Nc was significant

Regression time of color effect on critical propositions of all worked proofs were not significant

Initial reading comprehension times of Color effect on critical propositions of No.1, 2 & 4 were significant

a. Boldface are Significant data

Result(4)

Table 4 The percentages of correctness of paper-pencil recall post tests

	color	uncolor	average	*F*
Item 1	96.25	94.67	95.48	0.26
Item 2	80.00	71.35	75.48	0.33
Item 3	75.00	74.00	74.52	0.01
Item 4	53.33	44.38	48.71	0.36

There is no color effect on Paper-pencil recall post-test

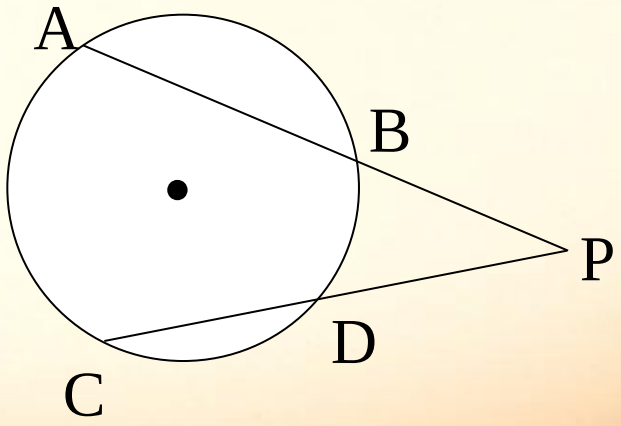
Conclusion and Discussion

- Figures are more important in reading comprehension of geometry proofs than other texts.
- Color cues affect the eye movement data of undergraduate students, but not the results of recall testing
  - Color figures affect the visualization and figure recognition which represents a decline in initial reading comprehension times
  - Color figures do not affect reasoning which represents similar regression time of two groups.
  - High cooperation and motivation of undergraduate students cause no significant difference of recall test between color and uncolor groups.

Thank you for your attention

已知：如右圖，兩弦 $\overline{AB}$ 與 $\overline{CD}$ 的延長線交於圓外一點P

求證：$\angle P = \frac{1}{2}\left(AC - BD\right)$



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證明：

1. $\angle 1=\angle P+\angle 2$ （三角形外角性質）…①

2. $\angle 1=\frac{1}{2}AC$ （圓周角）… ②，$\angle 2=\frac{1}{2}BD$ （圓周角）… ③

3. 將 ②、③ 代入① 知

$$\frac{1}{2}AC=\angle P+\frac{1}{2}BD$$

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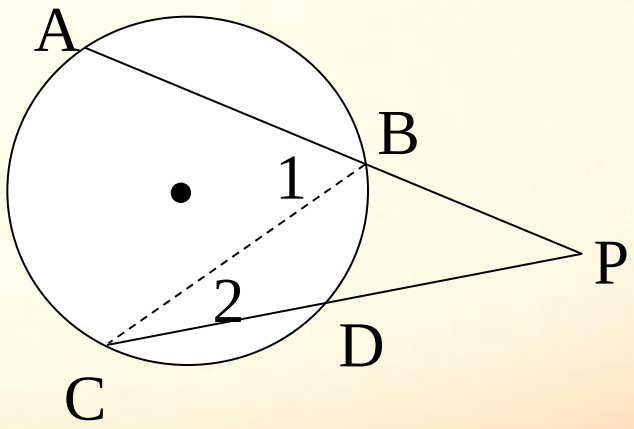

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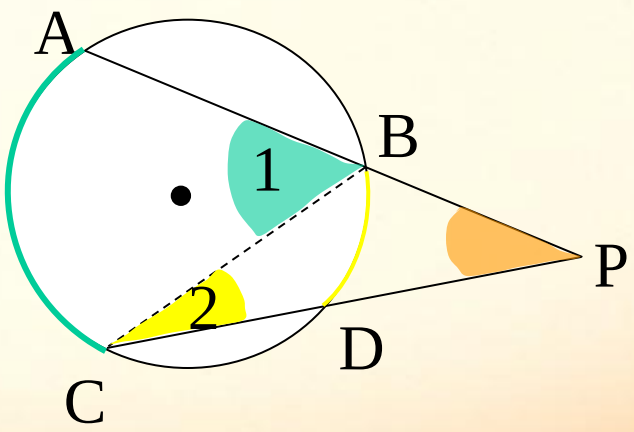

已知：如右圖，圓的兩弦 $\overline{AB}$ 與 $\overline{CD}$ 相交於P 點

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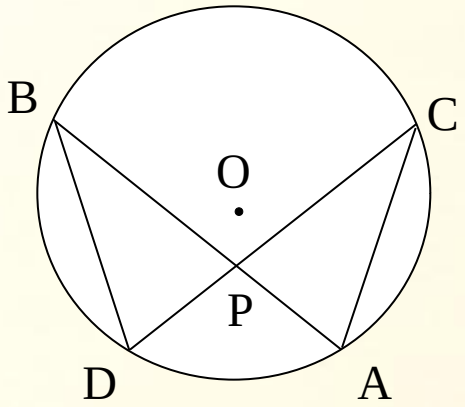

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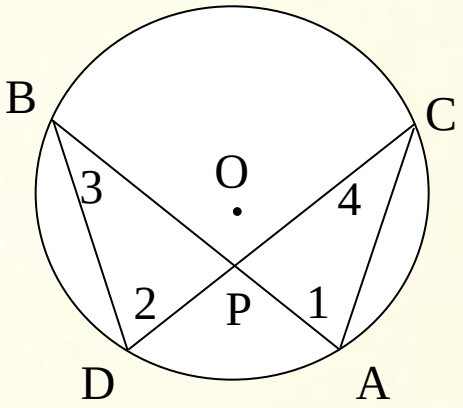

證明：

1. △PAC與△PDB中

   $\because\angle 1=\angle 2$

   $\angle 3=\angle 4$ （同一弧所對之圓周角相等）

   $\therefore$△PAC～△PDB（AA相似）

2. $\because$△PAC～△PDB

   $\therefore$ $\overline{PA}:\overline{PC}=\overline{PD}:\overline{PB}$

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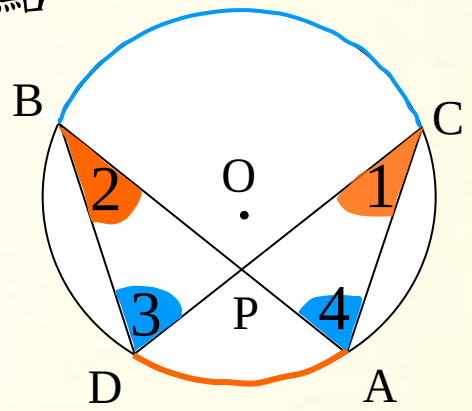

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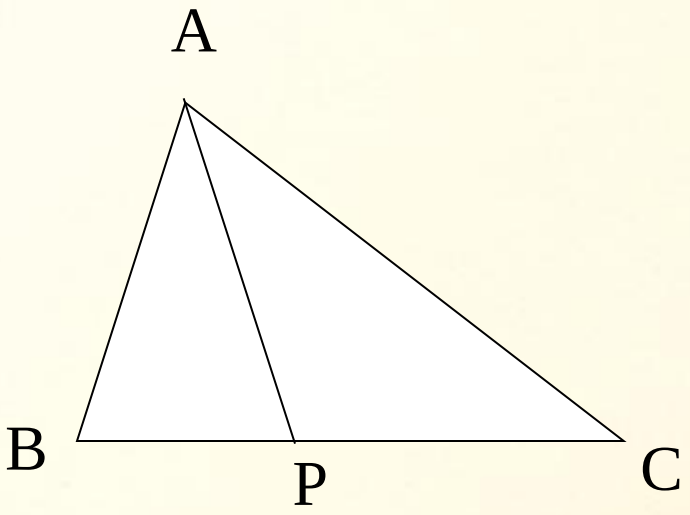

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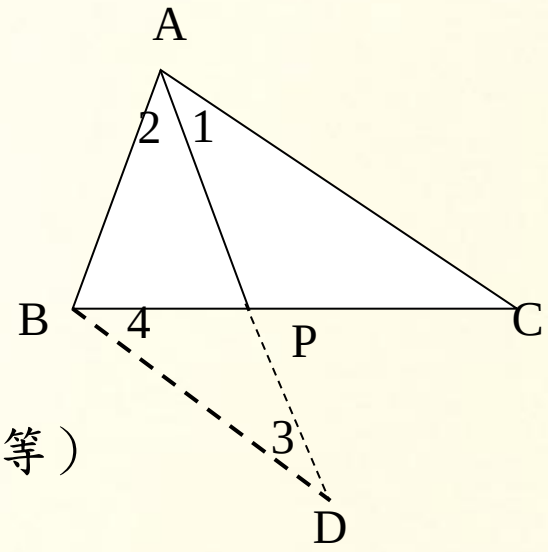

證明：

1. 延長$\overline{AP}$，在$\overline{AP}$上取一點 D，使$\overline{BD}//\overline{AC}$

   ∵ $\overline{BD}//\overline{AC}$，$\angle C=\angle 4$

   $\angle 1=\angle 3$ …①（內錯角相等）

   ∴△APC～△DPB（AA相似）

2. ∵△APC～△DPB

   ∴ $\overline{AC}:\overline{PC}=\overline{DB}:\overline{PB}$ …②（相似三角形，對應邊成比例）

3. △ABD中，∵ $\overline{AP}$ 平分$\angle BAC$，∴$\angle 1=\angle 2$…③

   由 ①、③ 知$\angle 2=\angle 3$，$\overline{AB}=\overline{BD}$ …④

   將 ④ 代入② 知 $\overline{AC}:\overline{PC}=\overline{AB}:\overline{PB}$

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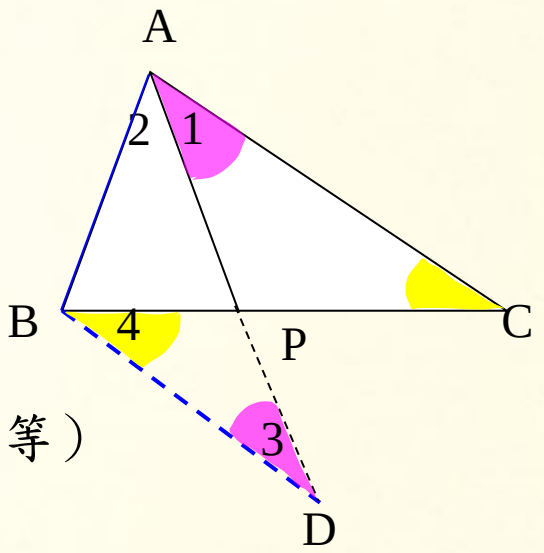

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Static Statement:

The upper left pulley is attached to the ceiling

Kinematic Statement:

The upper left pulley turns counterclockwise

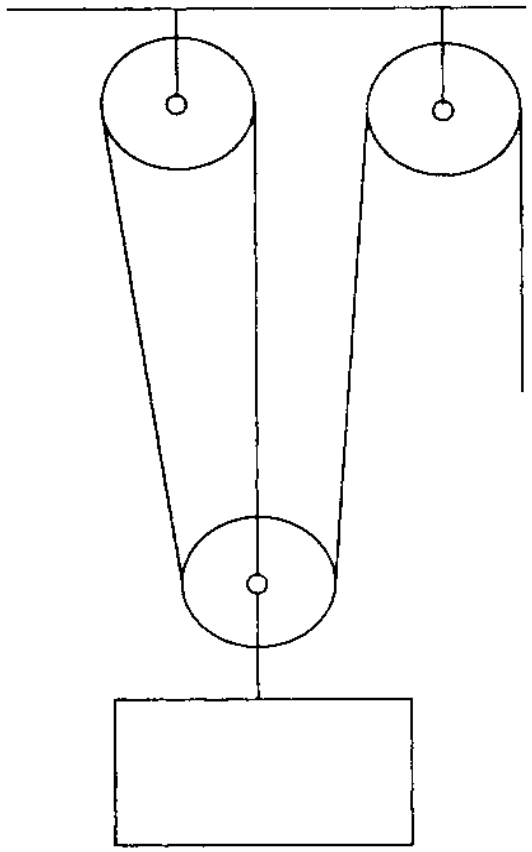

Figure 2. Pulley System 2 and sentences describing static and kinematic relations among the system components.