The influence of reading figures in geometry proof on eye movement

Authors: Tsai-Wen Lin, Chao-Jung Wu, & Sommer, S.

Presenter：Tsai-Wen Lin (PhD student)

National Normal Taiwan University

Introduction

§ Background

1. The importance of reading comprehension for learning geometry and geometry problem solving

2. The role played by attached figures in reading geometry proofs

3. The limitation of paper-and-pencil tests and interviews

4. The application of eye-related behaviors for mathematics education

Introduction

§ Purpose

1. To explore the role of attached figures for reading geometry proofs

2. To understand the cognitive processes in reading geometry proofs

Literature Review

§ Theory concerning geometry reading

Duval (1998,1999) contended that geometry utilizes three kinds of cognitive processes: **visualization**, **construction** and **reasoning**.

## § Eye movement of reading text with both text and figures

1. The average percentage of total fixation time
   This refers to the percentage of reading time on figures. Research results this percentage is from 6%-29% in text such as scientific text, ad. text, etc.
   (Hannus & Hyönä, 1999; Jian & Wu, 2012; Rayner, Caren, Rotello, & Stewart, 2001)

6.06 seconds

2.23 seconds

$2.23/(6.06+2.23)=27\%$ the average percentage of total fixation time
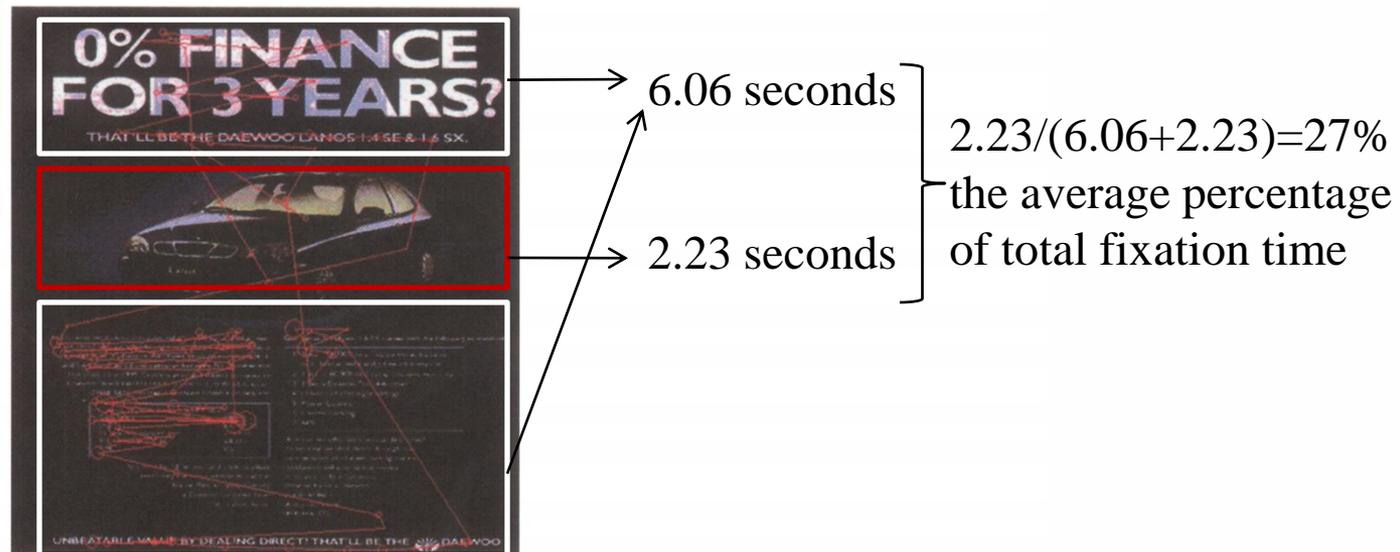
2. Number of saccades referring to figures

The role of eye movements in the comprehension of integrated layouts with text and figures has been explored in scientific text, advertisement, computer science, and online reading. (Jian & Wu, 2012; Schmidt-Weigand et al, 2010). In our study, it represents the eye movement between text and figures.
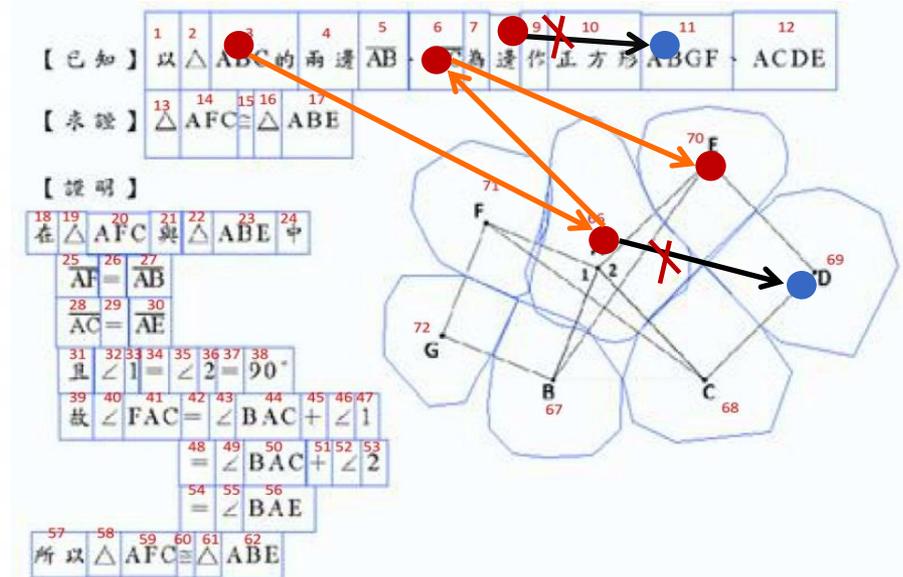
§ Eye movement of reading geometry proofs

Epelboim and Suppes (2001) studied eye movement models and visual working memory during problem solving in geometry. They contended that readers performed **eye movement between text and figures in order to construct the mental images of figures in their working memory.**

Numbers represent the order of eye movement from text to figure.
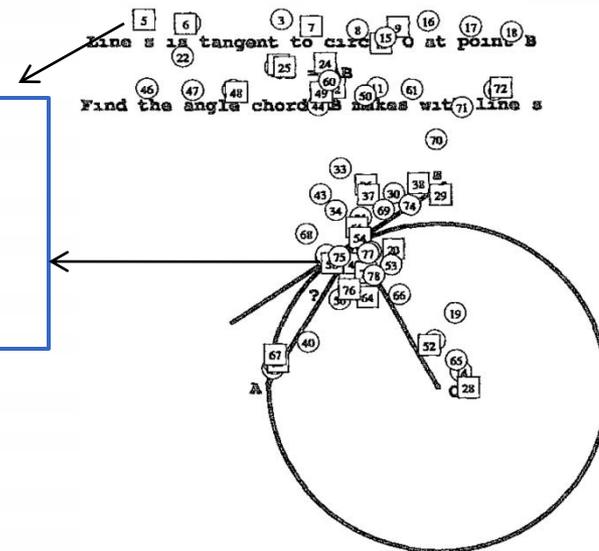
§ **Brief conclusion from literature review**

1. Geometric reading comprehension is composed of **visualization** as well as **reasoning**.

2. Figures play a significant part in reading geometry text for **using information** and **constructing mental models.**

3. **The average percentage of total fixation time** on figures is necessary to confirm the importance of the figures.

4. **Number of saccades** referring to figures is necessary to analyze cognitive processes in reading contents of geometry proofs.

Method

§ **Participants**

Sixty-five non-expert undergraduate students were selected from universities in Taipei. After exclusion of participants majoring in Mathematics, passing calibration, and validation procedures, the valid sample was fifty.

§ **Materials**

Layout was arranged in a formal way (shown as Fig 1).

The square item is simpler than the circle.

【已知】以△ABC的兩邊 $\overline{AB}$、$\overline{AC}$為邊作正方形ABGF、ACDE　　【已知】圓O的兩弦 $\overline{AB}$、$\overline{CD}$相交於P點

【求證】△AFC≅△ABE　　　　　　　　　　　　　　　　　　　　【求證】$\overline{PA} \times \overline{PB} = \overline{PC} \times \overline{PD}$

【證明】

在△AFC與△ABE中

　$\overline{AF} = \overline{AB}$

　$\overline{AC} = \overline{AE}$

　且∠1 = ∠2 = 90°

　故∠FAC = ∠BAC + ∠1

　　　　 = ∠BAC + ∠2

　　　　 = ∠BAE

所以△AFC≅△ABE

【證明】

在△PAC與△PDB中

　∠A = ∠D

　∠B = ∠C
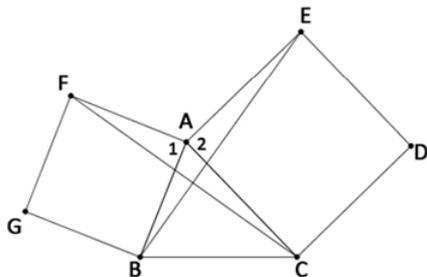
所以△PAC∼△PDB

因此 $\overline{PA} : \overline{PD} = \overline{PC} : \overline{PB}$
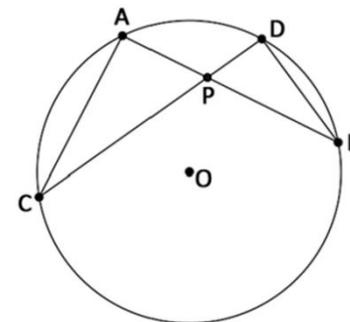
Square Item   **Circle Item**

Figure 1

§ **Apparatus**
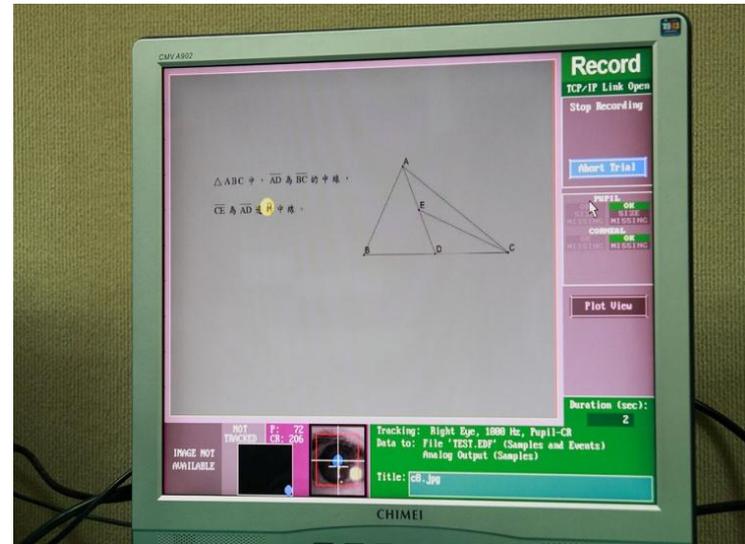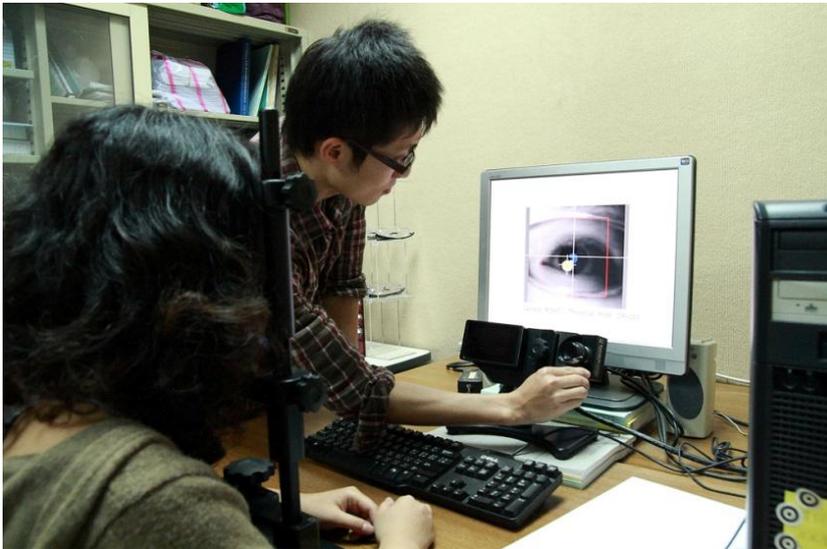
§ **Procedures**
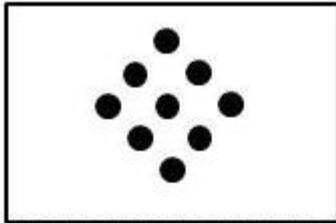
calibration

Item

Item with worked proof
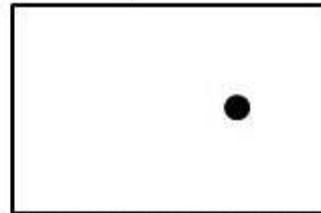
two items

calibration

§ **Data selection and analysis**

1. Outliers of reading-time and imprecise gazes were deleted.

2. Gazes apart from the calibration point were removed by checking each eye tracker, so that the final valid sample was fifty.

3. Data sets of 50 participants were analyzed. Each participant viewed two items. 100 eye trackings were collected.

§ **Data selection and analysis**

4.Contents of the text were divided into a variety of "Areas of Interests (AOIs)" based on the CKIP system developed by Taiwan's Academia Sinica (shown as Fig2).

The number of saccades appearing when participants read **mathematical symbols** located in the AOIs within "given" and "worked proof" was analyzed .
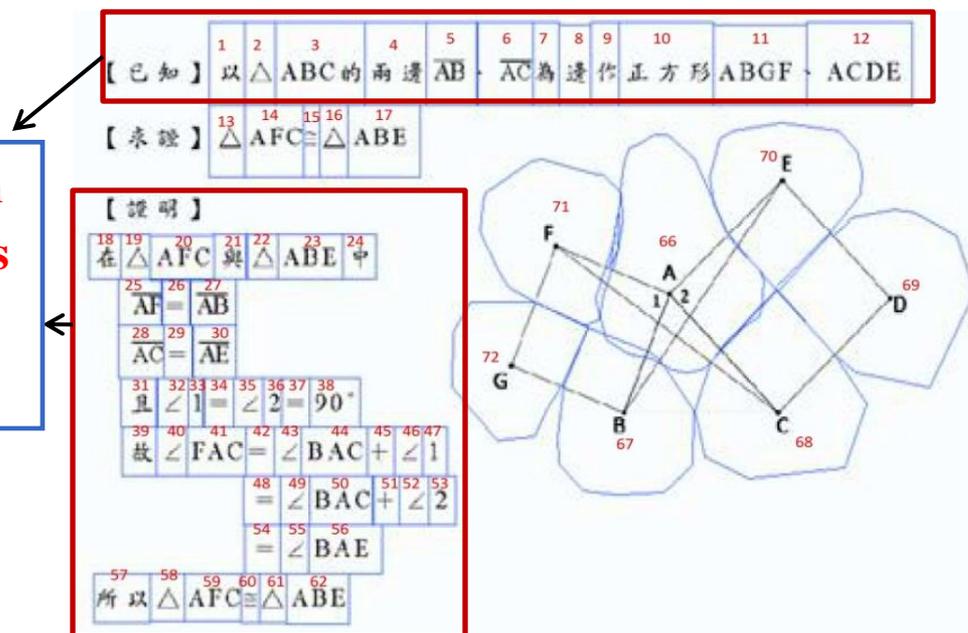
Fig 2 AOIs of two items

Results

§ Eye movement indices are shown in Table 1

	Square M(SD)	Circle M(SD)	F-value
Percentage of total fixation time on figures(%)	54	47	$F(1,49) = 25.11^{*}$
Number of saccades	20.74(8.65)	20.67(7.80)	$F(1,49) = 0.18$

1. First, the average percentage of total fixation duration in reading figures was 50% higher than that of other text (ex: scientific text, etc.) mentioned above.
2. Second, the number of saccades referring to figures differed non-significantly between two items, although specific eye movements will be explored and discussed in the following section.

Descriptive data of the number of saccades of given and worked proof for the two items is shown in Table 2.

proofs items	given M (SD)	worked proof M (SD)
Square	.51 (.37)	.51 (.25)
Circle	.45 (.36)	.72 (.32)

This reveals that more eye movement between text and the figure of Circle item is needed when participants are reading the more difficult item.

Table2 Descriptive data of the number of saccades referring to figures

1. The results of the within-subject analysis of variance revealed that the interaction was significant for stages and items, $F(1, 49) = 17.87, p < .001$.
2. Simple main effects occurred in the difference between the given and the worked proof of the Circle item.

Via analysis of recorded eye trackings, thirty-eight data sets were found to have the goal-driven pattern, representing the first gaze fixation on "proof" and the refocus on "proof" after reading the conclusion.
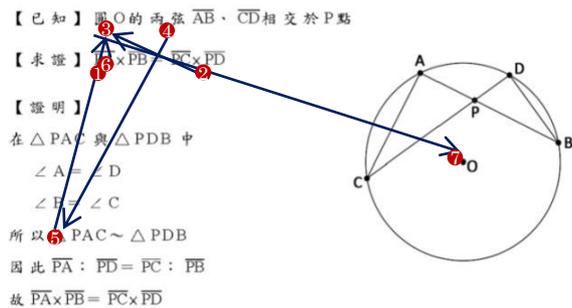
paths of eye movement
gazes

the AOI of proof
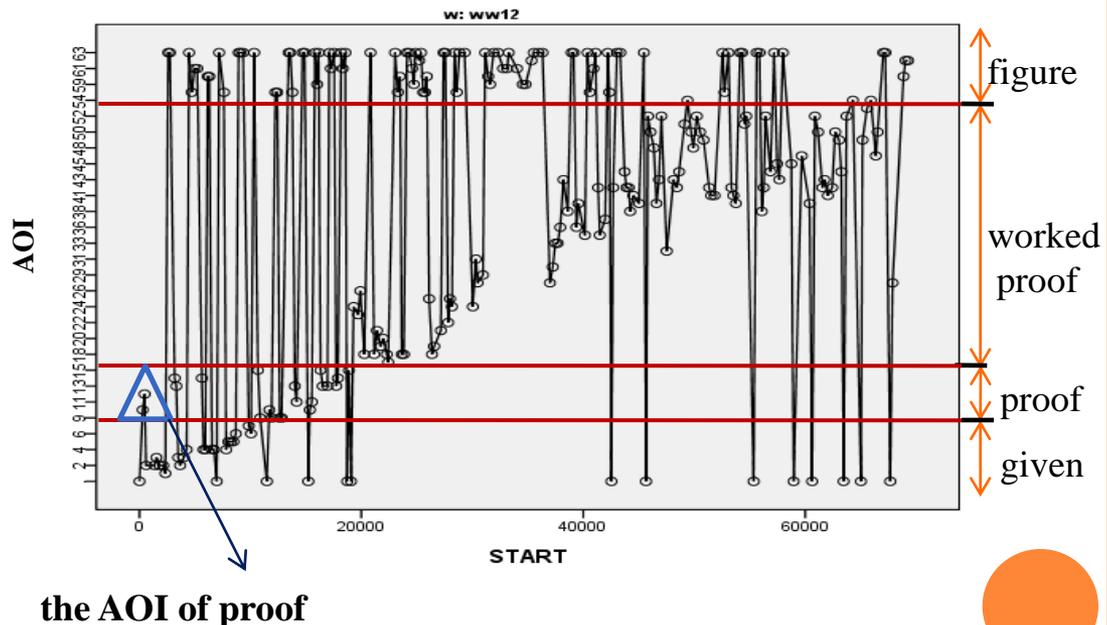
Discussion

§ Discussion

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┌─────────────────────┐     ┌─────────────────────┐     ┌──────────────────────────────────┐
│  text-driven reading │     │        given        │     │           worked proof            │
│ ┌───────────────────┐│  →  ├─────────────────────┤  →  ├──────────────────────────────────┤
│ │goal-driven reading││     │  visual perception  │     │ visual perception + visualization │
│ └───────────────────┘│     └─────────────────────┘     └──────────────────────────────────┘
└─────────────────────┘
```

eye movement

┌─────────────────────┐   ┌──────────────────────────────┐
│ number of saccades │   │ the higher average percentage │
│ referring to figures │   │ of total fixation time on figures │
└─────────────────────┘   └──────────────────────────────┘

1. to keep spatial representations in working memory

2. to b **This is apparent in text with different levels of difficulty.**
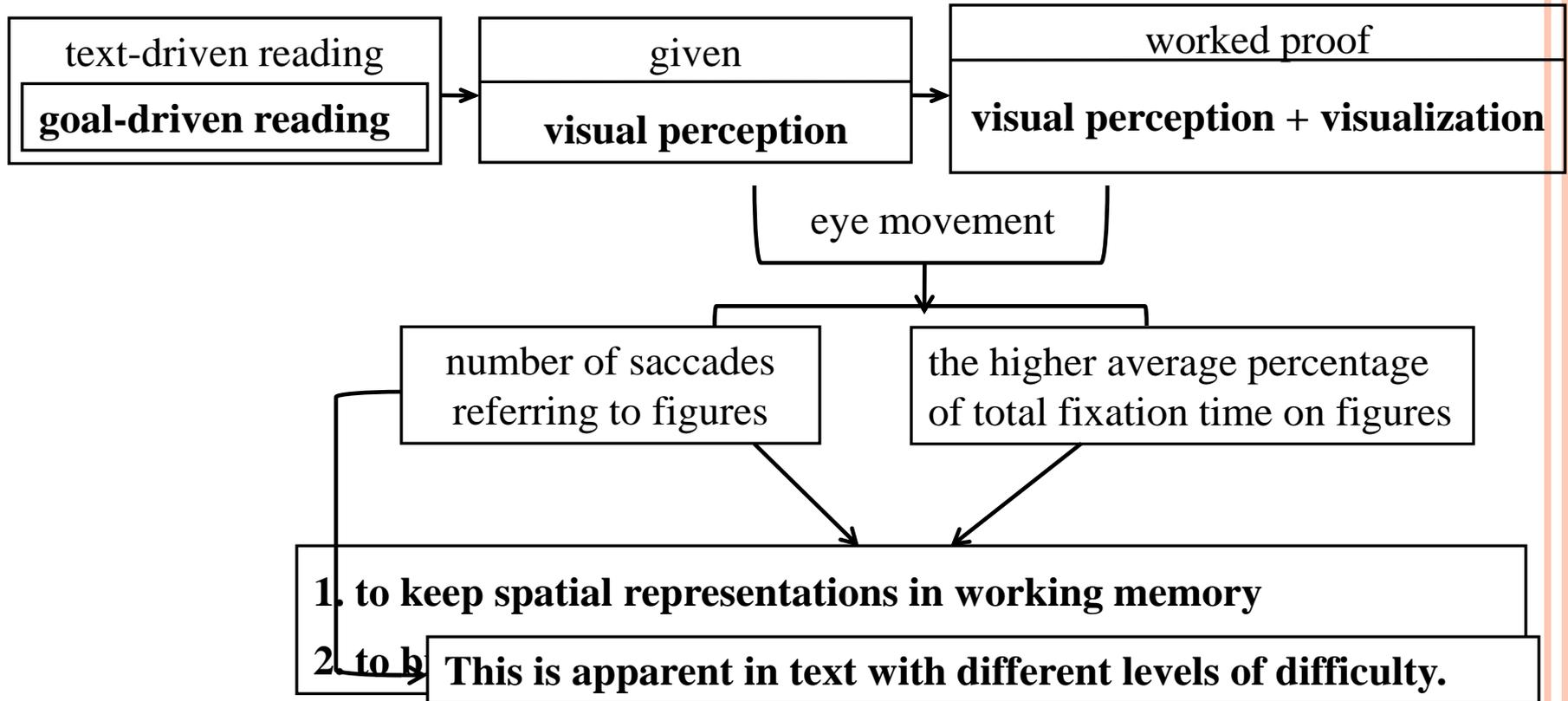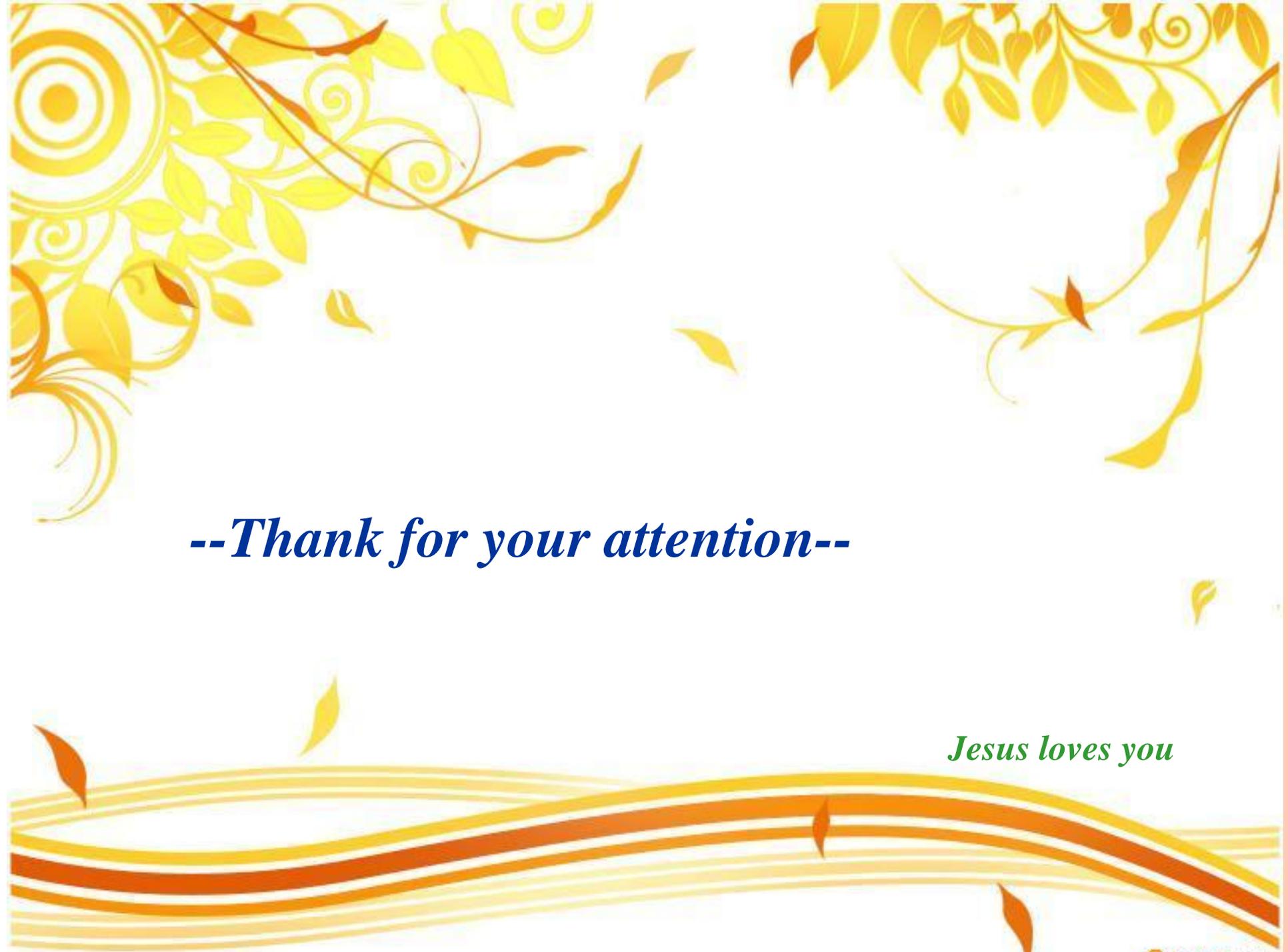
--Thank for your attention--

Jesus loves you